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The relativistic fluid equations governing large gravitationally collapsing systems can exhibit density singularities. Under asymmetric collapse, these same equations may support finite time relativistic velocity singularities ($v \to c$). These relativistic velocity singularities might contribute as a novel acceleration mechanism in such astrophysical systems as highly relativistic polar jet outflows and energetic cosmic rays. This paper provides a necessarily preliminary treatment due to the intractable nature of the problem.

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I. INTRODUCTION

The relativistic fluid equations describing gravitationally collapse can exhibit density singularities [1]. Necessarily then, the non-relativistic and small systems limit of these same equations also support density singularities though compressive forces other than gravity must be employed. For example, potential density singularities ($\rho \to \infty$) exist in shock tunnels [2] and in the hand-held Diamond-Anvil generating 2 million atmospheres of pressure [3], in cavitation generating energy densities sufficient to pit steel [4], in sonoluminescence generating temperatures in excess of 100,000 degrees [5–7], in explosive compression of magnetic fields generating fields up to 1,000T [8,9] and in supernovae collapse experiments where inertial confinement generates temperatures in excess of 100 million degrees [10].

A singularity results when a mathematical regular singular point is embedded within the governing equations, and such singular points are expected to survive the transition from the relativistic to non-relativistic equations, and visa versa. Indeed, it would be surprising if they did not. Then, the existence of velocity singularities ($v \to \infty$) in the non-relativistic fluid equations strongly suggests that relativistic velocity singularities ($v \to c$) should appear in the relativistic fluid equations. Here, c is Einstein's constant, the speed of light. This paper examines this possibility.

Non-relativistic velocity singularities appear when a fluid drop fissions in two [11–14], as thin-films are pinched to zero thickness [15,16], and in supersonic fluid jets expelled from squeezed systems [17], while finite time current singularities appear in magnetohydrodynamic models of magnetic reconnection [18].

Relativistic velocity singularities might contribute to asymmetric (or turbulent) supernovae collapse where bullets or jets of core material are expelled faster than and through overlying stellar layers [19,20], to magneto-hydrodynamic polar jets launched from accretion disks [21,22], and to cosmic ray acceleration generating maximum particle energies up to 10^{20}eV [23], while knowledge of velocity singularity time dependencies might inform studies of polar jet shock structures [24–26] as a time dependent source allows faster ejecta to overtake material previously ejected to generate shocks.

Multi-dimensional relativistic fluid systems are analytically and numerically difficult (see for example [27–32]). In the non-relativistic regime, analytic, numeric and experimental techniques all fail at singular points and these failures are expected to carry over to the relativistic limit. This paper then makes many simplifying assumptions to allow progress.

II. GENERAL VELOCITY SINGULARITIES

Velocity singularities are a common feature of typical fluid systems subject to compression, and their very pedestrian nature is best illustrated by considering fluid compression using the usual non-relativistic continuity equation

$$\partial_t \rho + \partial_x (\rho v_x) + \partial_y (\rho v_y) = 0 \tag{1}$$

describing fluid density ρ with velocities (v_x, v_y) in directions (x, y) [33,34]. (∂_α expresses rates of change in coordinate α.) A compression singularity can be modeled by a piston closing vertically in the y direction with negative (non-zero) velocity differentials $\Delta(\rho v_y) < 0$ appearing over shrinking displacements $\Delta y \to 0$. A fully enclosed piston lacks escape flows ($v_x = 0$) giving

$$\partial_t \rho = -\frac{\Delta(\rho v_y)}{\Delta y} \to \infty \tag{2}$$

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with singular solution $\rho(t) \to \infty$ in the limit $\Delta y \to 0$. These piston-type equations adequately model systems featuring either constraining side walls or spherical symmetry so escape velocities are zero during the collapse phase.

In contrast, velocity singularities appear when side-walls are absent or when asymmetric compression allows a non-zero escape velocity $v_x \neq 0$ described by

$$\partial_t \rho + \partial_x (\rho v_x) = -\frac{\Delta (\rho v_y)}{\Delta y} \ \to \ \infty. \tag{3}$$

The singular solutions are then either $\rho(t) \to \infty$ and/or $v_x(t) \to \infty$ in the limit $\Delta y \to 0$. These solutions then model high speed jets expelled perpendicular to applied squeezing forces.

Squeezing an arbitrary non-relativistic fluid introduces regular singular points into the governing equations which can force singular solutions. Such points are likely to be preserved in going over to the relativistic equations in the appropriate limit making it likely that squeezing a relativistic fluid will also introduce regular singular points into the governing equations. This is considered in the next section.

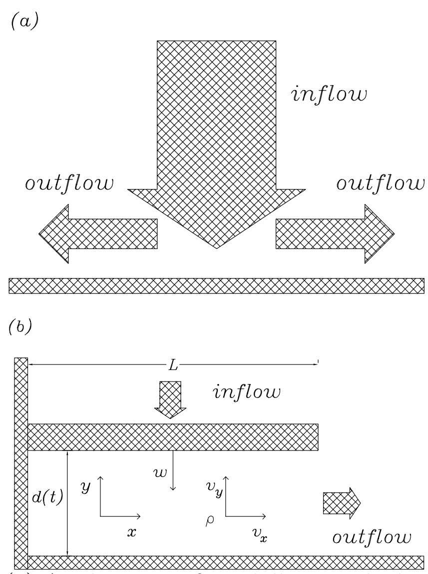

FIG. 1. (a) A schematic of a tidally stretched infalling column of perfect gas impacting an incompressible surface, and (b) modeling the column as a piston compressing underlying layers of gas noting parameters required for analytical treatment.

III. RELATIVISTIC VELOCITY SINGULARITIES

This section presents a simple physical model generating relativistic velocity singularities, incorporating many simplifying approximations so no strong claim is made that this model is realizable. Figure 1 schematically shows a tidally stretched column of a perfect gas impacting the incompressible surface of a neutron star, with the column being much taller vertically (of order hundreds of meters) than wide horizontally (of order tens of meters). The column is gravitationally accelerated to a substantial fraction of the speed of light and is rapidly decelerated close to the surface (over distances of order millimeters). Shocks are ignored. The impacting fluid cannot rebound vertically due to the infalling column and it is expected that ejecta travels horizontally along the (gravitational equipotential) surface at high speed. The impacting column can be crudely modeled as an open-sided piston descending vertically along the $-y$ axis to drive an expelled flow in the x direction.

This model ignores viscosity, heat conductivity, magnetohydrodynamic forces, radiative ionization, particle creation and destruction and impact effects on the neutron star where exposed neutronic matter probably begins to convert to atomic densities. Lastly, the model considers infalling matter only in the region of a few meters above the surface of the neutron star so that, in this localized region, gravitational effects can be ignored allowing the model to treat only a flat spacetime.

Consider a flat space-time with metric $\eta_{\alpha\beta} = \eta^{\alpha\beta} = \mathrm{diag}\,(-1,1,1,1)$ where primed coordinates $x'^{\alpha} = (t', x', y', z')$ give the contravarient velocity vector of the infalling fluid as $U'^{\alpha} = (\gamma, \gamma \mathbf{v}')$ with $\mathbf{v}' = (v_{x'}, v_{y'}, v_{z'})$ and $\gamma = 1/\sqrt{1 - v'^2}$ in geometrized units setting the speed of light to unity. (Greek indices run over values 0 — 4 while Latin indices vary from 1—3.)

A perfect gas has particle flux and energy-momentum tensor of

$$\begin{aligned} N'^{\alpha} &= n U'^{\alpha} \\ T^{\alpha\beta} &= \eta^{\alpha\beta} p + (p + \rho)\, U'^{\alpha} U'^{\beta} \end{aligned} \tag{4}$$

where n, p and ρ are the momentarily comoving frame particle number density, pressure and energy density respectively [35]. Conservation equations are then $N'^{\mu}{}_{;\mu} = 0$ and $T'^{\mu\nu}{}_{;\mu} = 0$ giving

$$\begin{aligned} D'\,(\gamma n) &= 0 \\ D'\left(\gamma^2 (p+\rho)\right) - \partial_{t'} p &= 0 \\ D'\left(\gamma^2 (p+\rho)\mathbf{v}'\right) + \nabla' p &= 0, \end{aligned} \tag{5}$$

with $D' = \partial_{t'} + \nabla'.\mathbf{v}'$ and $\nabla' = (\partial_{x'}, \partial_{y'}, \partial_{z'})$. These equations give respectively, particle number conservation, mass continuity and momentum conservation.

Moving to co-moving coordinates gives insight into the physical system, and we select a fluid element at some arbitrary height $y' = d_0$ at time $t' = 0$ which falls to $y' \approx 0$ near the surface due to compression by the overlying material. Then, its vertical coordinate is $y'(t') = f(t') d_0$ using a monotonically decreasing function $f(t')$ with $f(0) = 1$, and its vertical velocity for $t' > 0$ is $v_y(t') = \dot{f}(t') d_0$. Fluid compression occurs in the limit $f \to 0$. In particular, compression from atomic to nuclear densities implies f asymptotes approximately to $f \to 10^{-8}$. The transformation to unprimed co-moving coordinates is then

$$x^\alpha = \begin{pmatrix} t \\ x \\ \chi \\ z \end{pmatrix} = \begin{pmatrix} t' \\ x' \\ y'/f(t') \\ z' \end{pmatrix}, \tag{6}$$

so effectively, $\chi = d_0$ for each fluid element and vertical compression in physical space $\Delta y' \to 0$ corresponds to a constant separation in computational space, $\Delta\chi$ constant. The vertical compression is fully captured in the function $f(t) = f(t')$, and its effects can be assessed by tracking this function in the computational space governing equations.

The comoving metric $g_{\mu\nu}$ and proper time $d\tau^2 = g_{\mu\nu}dx^\mu dx^\nu$ are

$$\begin{aligned} d\tau^2 = &\left[\left(\dot{f}\chi\right)^2 - 1\right] dt^2 + 2f\dot{f}\chi dt\, d\chi \\ &+dx^2 + f^2 d\chi^2 + dz^2, \end{aligned} \tag{7}$$

while the non-zero affine connections are

$$\Gamma^\chi_{tt} = \frac{\ddot{f}}{f}\chi; \qquad \Gamma^\chi_{\chi t} = \Gamma^\chi_{t\chi} = \frac{\dot{f}}{f}. \tag{8}$$

As required for a flat spacetime, the Ricci tensor is zero everywhere (as also, the metric tensor $g_{\mu\nu}$ has three positive and one negative eigenvalue). The transformed contravariant velocity vector is $U^\alpha = [\gamma, \gamma\,(v_x, v_\chi, v_z)]$ with

$$v_\chi = \frac{\left(v_{y'} - \dot{f}\chi\right)}{f}. \tag{9}$$

Then, as required, the co-moving fluid has $v_{y'} = \dot{f}\chi (\equiv \dot{f}d_0)$ so that $v_\chi = 0$.

In computational space, the particle flux and energy-momentum tensors become

$$\begin{aligned} N^\alpha &= nU^\alpha \\ T^{\alpha\beta} &= g^{\alpha\beta}p + (p+\rho)\,U^\alpha U^\beta \end{aligned} \tag{10}$$

giving the computational space conservation equations $N^\mu{}_{;\mu} = 0$ and $T^{\mu\nu}{}_{;\mu} = 0$ as

$$\begin{aligned} D\,(f\gamma n) &= 0 \\ D\left(f\gamma^2(p+\rho)\right) - \partial_t(fp) &= -\dot{f}\partial_\chi(\chi p) \\ D\left(f\gamma^2(p+\rho)\mathbf{v}\right) + \nabla\,(fp) &= \begin{pmatrix} 0 \\ A \\ 0 \end{pmatrix}, \end{aligned} \tag{11}$$

with

$$\begin{aligned} A = &-\ddot{f}\chi T^{tt} - 2\dot{f}T^{t\chi} - \partial_t\left(\dot{f}\chi p\right) \\ &-\partial_\chi\left[fp\left(\frac{1}{f^2} - \frac{\dot{f}^2\chi^2}{f^2} - 1\right)\right] \end{aligned} \tag{12}$$

and $D = \partial_t + \nabla.\mathbf{v}$, $\nabla = (\partial_x, \partial_\chi, \partial_z)$ and

$$\begin{aligned} T^{tt} &= \gamma^2(p+\rho) - p \\ T^{t\chi} &= \gamma^2(p+\rho)v_\chi + \frac{\dot{f}}{f}\chi p. \end{aligned} \tag{13}$$

These equations give respectively, particle number conservation, mass continuity and momentum conservation as can be seen by taking the limit $f = 1$ and $\dot{f} = 0$.

The principle feature of these equations is the linkage everywhere on the left-hand-side (LHS) between the squeezing parameter $f(t)$ and γ. As is well known, the singularity $\gamma \to \infty$ ensures that all fluid velocities remain less than the speed of light, $v < c$. Then, the countervailing linkage between this singularity and the zero asymptote of the squeezing parameter $f \to 0$ suggests a minimization of the effects of the combined terms $f\gamma$ and $f\gamma^2$. If for example, $f \approx 10^{-8}$ as when atomic matter is compressed to nuclear densities, γ could increase by a factor of approximately 10^4 over what could otherwise be achieved in a given fluid system.

The physical meaning of the squeezing parameter f can be seen in more detail using the injection form of the relativistic fluid equations

$$\begin{aligned} D\,(\gamma n) &= -\frac{\dot{f}\gamma n}{f} \\ D\left(\gamma^2(p+\rho)\right) - \partial_t p &= -\frac{\dot{f}}{f}\partial_\chi(\chi p) \\ &\quad -\frac{\dot{f}}{f}\left(\gamma^2(p+\rho) - p\right) \\ D\left(\gamma^2(p+\rho)\mathbf{v}\right) + \nabla p &= \begin{pmatrix} 0 \\ A/f \\ 0 \end{pmatrix} - \frac{\dot{f}\gamma^2(p+\rho)}{f}\begin{pmatrix} v_x \\ v_\chi \\ v_z \end{pmatrix}. \end{aligned} \tag{14}$$

The left hand sides (LHS) here are identical to those of Eq. (5) while the right hand sides (RHS) represent sources injecting matter and momentum into the computational space, including into the horizontal directions x and z. (Similar equations are used to describe rockets employing real injection systems to generate high speed expelled flows [33].) The injection sources are dominated in the relativistic large γ regime by terms proportional to the ratio $\dot{f}/f$ and either γ or γ^2. Then, as long as $\dot{f} \neq 0$, the respective limits $\gamma \to \infty$ and $f \to 0$ reinforce each other to inject (potentially) infinite mass and momentum in finite time to drive a horizontal velocity singularity.

A very approximate analysis of the velocity and energy of a horizontal expelled jet can be performed but solutions are suggestive only. Consider the case where the fluid is vertically co-moving so that $v_\chi = 0$ ($v_y = -\dot{f}\chi$) and possesses near relativistic horizontal velocity v_x constituting an expelled jet. (Symmetry allows ignoring the z direction.) Fluid velocities are then $(v_x, 0, 0)$. In the large γ limit each of the above conservation equations gives the approximate time rate of change of expulsion velocity as

$$\partial_t v_x \approx O\left(\frac{1}{\gamma^2}\right) - \frac{\dot{f}}{f\gamma^2 v_x}$$
$$\partial_t v_x \approx O\left(\frac{1}{\gamma^2}\right) - \frac{\dot{f}\left(\gamma^2(p+\rho)-p\right)}{2f(p+\rho)\gamma^4 v_x}$$
$$\partial_t v_x \approx O\left(\frac{1}{\gamma^2}\right) - \frac{\dot{f} v_x}{f(1+2\gamma^2 v_x^2)} \tag{15}$$

where the top line results from the conservation of particle number, the second line from the conservation of mass and the third line stems from the conservation of momentum in the x direction. The generic form of these equations in the large γ limit is

$$\partial_t v_x \approx O\left(\frac{1}{\gamma^2}\right) - \frac{\dot{f}}{af\gamma^2 v_x} \tag{16}$$

where $a = 1$ or 2 and this form is used for further discussion. The first term on the RHS is independent of f and, in the absence of squeezing ($\dot{f} = 0$), necessarily goes to zero so that $\partial_t v_x \to 0$ as $\gamma \to \infty$ ensuring $v < c$. The second term reflects squeezing effects and is positive ($\dot{f} < 0$) and the linkage between the limits $f \to 0$ and $\gamma \to \infty$ offsets the effect of γ. This equation can be integrated by ignoring the small first term with initial conditions $f(t_0) = 1$ and $v_x(t_0) = v_0$ giving

$$v_x^2(t) = 1 - f^{2/a}\left(1 - v_0^2\right), \tag{17}$$

or equivalently

$$\gamma^2(t) = \frac{\gamma_0^2}{f^{2/a}}. \tag{18}$$

The function $f(t)$ is arbitrary and we consider the non-physical but insightful limit of full closure, $f(T) = 0$ for some time $T > t_0$ giving

$$v_x(T) = 1, \tag{19}$$

a light speed jet. It is of course impossible for any material fluid to reach light speed though squeezing might approach this ideal using singular mass and momentum injection source terms proportional to $-\dot{f}\gamma^2/f$.

Particle energy is then

$$E \propto \gamma(t) \propto \frac{1}{f^a} \to \infty, \tag{20}$$

though this result is suggestive only.

IV. NONRELATIVISTIC VELOCITY SINGULARITIES

For completeness, details of the non-relativistic velocity singularity created by asymmetrically compressing a fluid are given. (See Ref. [17].) The conservative form of the inviscid and dimensionless Euler equations with zero conductivity are derived from the the relativistic equations (11) after taking the small velocity limit $\gamma \to 1$, $p \approx O(\rho v^2)$, and $p + \rho \to \rho$ so now ρ and p become the usual fluid density and pressure. The use of comoving coordinates [Eq. (6)] gives the Energy-Momentum tensor for a perfect gas as

$$T^{00} = \rho$$
$$T^{0j} = \rho v^j$$
$$T^{ij} = \rho v^i v^j + \mathrm{diag}\left(p, p/f^2, p\right). \tag{21}$$

Here, the diagonal pressure terms in the vertical direction are scaled by a factor $1/f^2$ over those in horizontal directions so that the limit $f \to 0$ creates the large pressure gradients driving the horizontal velocity singularity. Conservation of the energy-momentum tensor $T^{\mu\nu}{}_{;\mu} = 0$ then gives the modified Euler equations

$$\partial_t U + \partial_x F + \partial_\chi G = 0 \tag{22}$$

$$U = \begin{pmatrix} f\rho \\ f\rho v_x \\ f\rho v_y \end{pmatrix} \qquad F = \begin{pmatrix} f\rho v_x \\ f\left(\rho v_x^2 + \rho^\gamma/\gamma\right) \\ f\rho v_x v_y \end{pmatrix}.$$
$$G = \begin{pmatrix} f\rho v_\chi \\ f\rho v_x v_\chi \\ f\rho v_y v_\chi + \rho^\gamma/\gamma \end{pmatrix}$$

Here $v_\chi = (v_y - \dot{f}\chi)/f$ and we show mass continuity (top line) and momentum conservation in the x and y directions. (Symmetry constrains fluid motions to this plane.) Adiabatic perfect gases with $p = \rho^\gamma$ ($\gamma = 1.4$) are considered and all variables are dimensionless with dimensioned (primed) variables being given by $x' = xL$, $v' = va_0$, $p' = pp_0$, $\rho' = \rho\rho_0$ and $t' = tL/a_0$ with L being some convenient length parameter and $a_0^2 = \gamma p_0/\rho_0$ giving the local speed of sound.

Analytic solutions can be obtained in the low velocity limit $v_x \ll 1$ and $v_x^2 = 0$, and by assuming linear squeezing $f(t) = 1 - t/T$ for some closure time T and a co-moving fluid $v_y = \dot{f}\chi$ and $v_\chi = 0$. The analytic solution for an incompressible fluid with $\rho = 1$ is

$$v_x(x,t) = -\frac{\dot{f}x}{f} \tag{23}$$

displaying a velocity singularity as $f \to 0$. This solution is strictly valid only while $v_x < 0.3$ where an unconfined compressible fluid remains approximately uncompressed.

V. CONCLUSION

This paper uses a preliminary and necessarily crude analysis to establish that relativistic velocity singularities can appear in relativistic hydrodynamic equations. Velocity singularities provide such an elegant and simple means of sourcing near light speed relativistic jets that it would be surprising if this mechanism was not accessed in some astrophysical systems.

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